TWO TASTY SERVINGS OF PI

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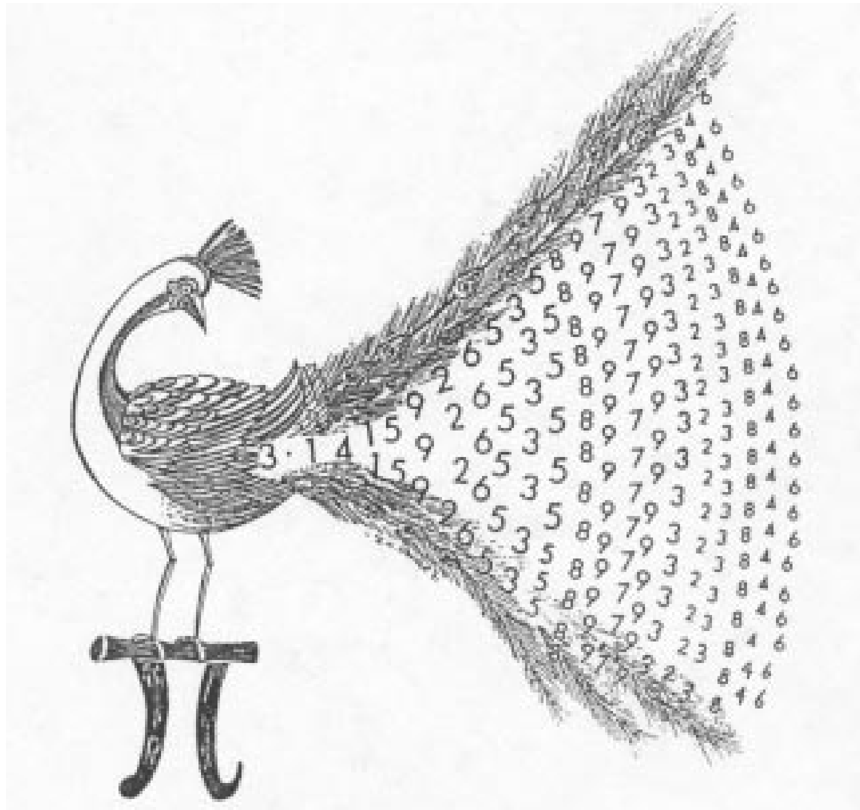
Pi: A Source Book
by Lennart Berggren, Jonathan Borwein and Peter Borwein. Springer, 1997, 716pp, \$37.50.
The Joy of π
by David Blatner. Allen Lane, The Penguin Press, 1997, 130pp, \$12.99.

Perhaps no concept has captured the mathematical imagination more than the circle ratio π, while no mathematical symbol has evoked more mystery, romanticism and popular appeal than π itself. Why this fascination with mathematics' most famous number? For the professional mathematician, π has long presented a challenge, being taciturn by nature and yielding up its splendours only grudgingly. When they are teased out, however, the effort expended is often handsomely repaid. Among reasons drawing the amateur to π are: its habit of turning up everywhere, often in unexpected places; the highly publicized search for its decimal digits, with world records tumbling almost annually in today's age of the supercomputer; and its long, colourful history filled with incident, drama, humour, genius and eccentricity. Continued interest in π over four millennia has resulted in the accumulation of a vast π-archive. What, then, do the two books under review here contribute to this collection?

Pi : A Source Book, the first source book on π ever to be published, documents, mainly through original writings, the history of π from the dawn of mathematical time to the present day. One only has to glance at the Contents to appreciate the pre-eminent role played by π in the history of mathematics, the seminal ideas to which it has given birth, and the number of illustrious mathematicians who have fallen under its spell. Furthermore, the list of titles supports the authors' claim that



the computation of π is the one topic from the most ancient stratum of mathematics that continues to be vigorously researched today, and that to trace its development is to follow a thread which winds through geometry, analysis and special functions, numerical analysis, algebra and number theory. One beauty of the anthology is the inclusion of so much contemporary, yet still accessible, mathematics - more than half of the collected articles are from the latter half of this century, the most recent published in 1996, the year of the book's completion!

The compilation itself comprises seventy articles (mostly research papers, but also a few historical studies and items of a more light-hearted nature) arranged chronologically and presented in their original form (photocopies, in fact) without accompanying comment, although each is accorded an illuminating one sentence description in the Contents. The introduction provides an overview of the whole collection, which serves to place individual articles into historical context, and there are three short appendices: *On the Early History of Pi*, *A Computational Chronology of Pi* (out of date even before publication) and *Selected Formulae for Pi*. A striking multicoloured design adorns the front cover, while more than 10,000 digits of π's decimal expansion decorate it on the inside.

The authors divide their material into three periods: before Newton, Newton to Hilbert, and the Twentieth Century. The earliest of these, containing fifteen papers from Egyptian, Chinese, Arabic, Indian and European sources, commences with a problem from the Rhind Papyrus (1650 BC) showing that the ancient Egyptians assumed an implicit value for π

of 256/81, and concludes with the debut of π to denote the circle ratio, in William Jones' *A New Introduction to the Mathematics* (1706). Incidentally, the Jones' extract can also boast the first calculation of π to 100 decimal places, *computed by the accurate and Ready Pen of the Truly Ingenious Mr John Machin*, but no mention is made of this. Archimedes' *On the Measurement of the Circle*, which dominated the subject in the pre-calculus era, is well represented, as are the original derivations of the first infinite expressions for π, those linked with the names of Viète (1593) and Wallis (1655). Another gem is Ranjan Roy's paper on the independent discovery of the power series for $\tan^{-1} x$ by Gregory (1671), Leibniz (1673) and a lesser known Indian mathematician, Nilakantha (1450).

What the period from Newton to Hilbert lacks in quantity, with only nine representative papers, it certainly compensates for in quality. Euler's dazzling mastery of formal algebraic manipulation, combined with innate good judgement, is exhibited in a chapter from his *Introduction to Analysis of the Infinite* (1748), which includes derivations of his celebrated series for powers of π. Then follow the first proofs of the transcendence of e, by Hermite (1873), and of π, by Lindemann (1882). Lindemann's paper, a landmark in the history of mathematics, showed once and for all that *the circle could not be squared.* Simpler proofs of the transcendence of π by Weierstrass (1885) and Hilbert (1893) are also given.

The twentieth century selections are divided between analytical and computational studies. Opening the former is Ramanujan's seminal paper *Modular Equations and Approximations to* π (1914), which exhibits some remarkable series for $1/\pi$. Watson's *The Marquis and the Land Agent: A Tale of the Eighteenth Century* (1933) is a delightful exposition of the early development of elliptic functions, which play a role in some modern computations of π. Other highlights include: Niven's one page proof of the irrationality of π, influential papers by Kurt Mahler and Alan Baker, and two articles on Apery's controversial proof (1978) of the irrationality of $\zeta(3)$. The computational selection covers the first electronic computation (ENAIC) of π in 1949, the independent discovery of arithmetic-geometric mean based algorithms for the computation of π by Salamin and Brent in 1976, and papers by Kanada, the Borwein brothers and the Chudnovsky brothers, today's leading exponents on the computation of π. A recent (1996) paper by David Bailey, Peter Borwein and Simon Plouffe serves as a worthy climax to this wonderful treasury and points the way to future developments. It describes a fast algorithm for determining *individual* digits of π in certain bases and illustrates its effectiveness by showing that the ten billionth hexadecimal digit of π is a 2 !

Few mathematics books serve a wider potential readership than does a source book and this particular one is admirably designed to cater for a broad spectrum of tastes: professional mathematicians with research interest in related subjects, historians of mathematics, teachers at all levels searching out material for individual talks and student projects, and amateurs who will find much to amuse and inform them in this leafy tome. The authors are to be congratulated on their good taste in preparing such a rich and varied banquet with which to celebrate π.

The Joy of π is a highly entertaining, lavishly designed book, which more than fulfils the expectation generated by its title and striking dust jacket blazoning an incandescent π shining forth from a star- studded jet sky. It is unashamedly popular in its approach, clearly aimed at the mass market, somewhat along the lines of the bestselling *Longitude* by Dava Sobel and the books on *Fermat's Last Theorem* by Aczel and Singh, but less substantial. In a lively and engaging style, the author tells the tale of π and man's fascination with it, sprinkling his narrative with rich helpings of π trivia: tidbits about π-eccentrics, π's own idiosyncrasies, multilingual mnemonics for pi, and pi-inspired quotations, poems, limericks, anecdotes, jokes and cartoon. In addition to more familiar stories, such as Indiana's notorious attempt to legislate a legal value of pi in 1897, there are others that are brand new, like a transcript from the OJ Simpson trial in which an FBI agent and the learned Judge express differing opinions on the value of pi, the former believing it to be 2.12, the latter 3.1214 ! Each page is individually and attractively, if occasionally over fussily, laid out with imaginative use of two-colour (black and green) artwork, although it is surely a misjudgement to squander space by strewing a million illegible decimal digits of π across the pages of the book, when most of them are infuriatingly unnumbered. Readers who still have not had their fill of pi are exhorted to start Web- surfing at http://www.joyofpi.com .

One item appearing in both books, and for the first time in print, is Michael Keith's *pi Mnemonics and the Art of Constrained Writing*. This has for its showpiece a rewriting of Edgar Allen Poe's poem, *The Raven* , in such a way as to preserve as far as possible the story, tone and rhyming scheme of the original, while simultaneously creating a 740 word mnemonic poem for π. In the *Source Book* the mnemonic begins as intended: *Poe E, Near a Raven* . . . , but in *The Joy of* π it commences *Pie E, Near a Raven* . . . , under the circumstances, a most forgivable Freudian slip!